H U B B L E
15 YEARS OF DISCOVERY

## TABLE OF CONTENTS

Exercise 1: How Hubble Made the Universe Larger ..... 3
Exercise 2: Weighing the Earth with Hubble. ..... 4
EXercise 3: When is a planet a planet? ..... 6
EXERCISE 4: AURORAE EVERYWHERE? ..... 7
Exercise 5: Space is Empty - and Huge! ..... 9
Exercise 6: Supermassive Black Holes Discovered ..... 10
Exercise 7: Scaling the Universe: Measuring Distances with Cepheid Variable Stars ..... 14
Exercise 8: Getting an Estimate of the Age of the Universe ..... 16
Teacher's Guide ..... 20


#### Abstract

t the beginning of last century it was commonly believed that our galaxy, the Milky Way, constituted the whole Universe and that the stars in the Milky Way were distributed throughout this Universe. The star nearest to us, our Sun, happened to be close to the centre of this Universe, or so it was thought. However, not everybody was convinced. Looking at the night sky with just the naked eye, most of what you see are stars, but not exclusively. From Earth's Southern Hemisphere, you can see large fuzzy patches called the Large and Small Magellanic Clouds. If you know where to look in the Northern Hemisphere, you can see also a dim fuzzy patch, called the Andromeda Nebula. Some thought that such clouds and nebulae were not part of our Milky Way galaxy. The conjecture was that they were in fact neighbouring galaxies, consisting of many stars, but located so far away that they appeared as fuzzy blobs. So finding the distance to these fuzzy blobs proved to be crucial in establishing which view of the Universe was correct. In the 1920s Edwin Hubble determined that the Andromeda Nebula was actually more than a thousand times further away than the stars we see in the sky. The Space Telescope is named in tribute to him as the man who is widely regarded as the founder of modern cosmology. Edwin Hubble proved that not all we see in the sky lies within our own local galaxy, the Milky Way, but that the cosmos extends far beyond. Today we know that the Universe is filled with billions and billions of galaxies instead of one.


- Exercise 1a: Describe in a few sentences how the people living around the time of the First World War thought the Universe looked.
- Exercise 1b: Describe in a few sentences how this had changed twenty years later, when Edwin Hubble had made his discoveries.

And, having established that the Universe was built on a wholly different scale, Edwin Hubble then went on to discover that this grand new Universe was also expanding. Just imagine the effect on the average person in a world where it was believed only recently that the Solar System was the centre of a quiet steady Universe.

The Hubble Space Telescope orbits the Earth with a period of 96 minutes at a height of 568 km above its surface. This information, along with some basic equations relating to circular orbits, allows us to "weigh" the Earth with the Hubble Space Telescope (assuming that Hubble's orbit is entirely circular ).

The force needed to hold an object in a circular orbit is generally described by this equation:

$$
\begin{equation*}
\mathrm{F}=\mathrm{mv}^{2} / \mathrm{r} \tag{1}
\end{equation*}
$$

where $m$ is the mass of the object (i.e. the telescope), $v$ is the velocity of the object and $r$ is the distance from the centre of the orbit (here the centre of the Earth) to the telescope.

Later you will need the orbital velocity of Hubble - how fast it has to travel to complete one orbit in the period - this is the $v$ in the equation above.

- Calculate the orbital velocity of Hubble (in the units of $\mathrm{m} / \mathrm{s}$ ).
- Exercise 2a: Find the total radius of Hubble's orbit in $m$, remembering that the Earth's radius is 6378 km , and Hubble orbits at a height of 568 km : $\qquad$ m
- Exercise 2b: Find the length of Hubble's orbit in $m$. You will need the formulae for the perimeter of a circle in terms of the orbit radius (in m ): $\qquad$ m
- Exercise 2c: Then divide by the time for one period (in s)
$\qquad$ s.
- Exercise 2d: Final result for orbital velocity: $\qquad$ $\mathrm{m} / \mathrm{s}$

Newton's law of gravitation states that the mutual gravitational force between two objects is given by:

$$
\begin{equation*}
\mathrm{F}=\mathrm{GMm} / \mathrm{r}^{2} \tag{3}
\end{equation*}
$$



Orbit of Hubble around the Earth (drawn to scale). Credit: ESA.
where $G$ is the gravitational constant, $6.67 \times 10^{-11} \mathrm{~m} 3 /\left(\mathrm{kg} \cdot \mathrm{s}^{2}\right), M$ is the mass of the larger object (here the Earth), $m$ is the mass of the smaller object (here the Hubble Space Telescope) and $r$ is the distance between the centres of the two objects.

Since Hubble stays in the same orbit these forces balance each other, i.e. they have the same numerical size.

- Exercise 2e: Let the two expressions (1) and (3) be equal, and derive an expression for the mass of the Earth in terms of G,r and v-all quantities that you know
$\qquad$
- Exercise 2f: Calculate the mass of the Earth using the expression you just derived.
$\qquad$ kg

Notice that the mass of the telescope was not actually needed to do this calculation. So your result holds for all satellites, whether it's a communications satellite or the Moon: the velocity is enough to determine the orbital height and the mass of the satellite is not important.

- How do your calculations compare with the accepted value for the mass of the Earth $5.976 \times 10^{24} \mathrm{~kg}$.

Exercise 2 g: Can you think of other methods to derive the mass of the Earth?

## EXERCISE 3:

WHEN IS A PLANET A PLANET?

Pluto (right) and its moon Charon (left) as observed by the Hubble Space Telescope. Credit: Dr. R. Albrecht, ESA/ESO Space Telescope European Coordinating Facility; NASA.

■rom the DVD you learnt:

Since Pluto's discovery in the 1930s, and its satellite Charon's in the 1970s, astronomers have tried to figure out if there's anything else out there, beyond the ninth planet.

In 2003, Hubble spotted something moving fast enough across the background offaraway stars to be an object within the Solar System. Estimates show that it could be about the size of a planet and it has been named Sedna, after an Inuit goddess.

Sedna may be 1500 km in diameter, that's about three quarters the size of Pluto, but it is so far away that it appears as just a small cluster of pixels even to Hubble. Nevertheless, it is the largest object discovered in the Solar System since Pluto.

Even though this discovery was made in 2003, we are still looking for the newspaper headlines saying, "Hubble discovers tenth planet in the Solar System".

Sedna and possibly also the planet Pluto are just two of a large number of icy objects that orbit the Sun out beyond Neptune. The area where these objects orbit is called the Kuiper belt, named after the Dutch astronomer Gerard Kuiper, who, in 1951, predicted the existence of a large number of icy objects in this location.

In 2002 another large object was discovered in the Kuiper belt. It is called Quaoar and has a diameter of about 1300 km .

Before answering the questions below you might want to do some background reading, for example at the following websites:

NASA's Solar System Exploration for KIDS: http://solarsystem.jpl.nasa.gov/planets/index.cfm
ESO/EAAE's Journey across the Solar System: http://www.eso.org/outreach/eduoff/edu-materials/info-solsys/

STSCI's Solar System Trading Cards: http://amazing-
space.stsci.edu/resources/explorations/trading/
Michael E. Brown's website on Sedna: http://www.gps.caltech.edu/~mbrown/sedna/

- Exercise 3a: What is the difference between a planet and a moon?
- Exercise 3b: Discuss reasons for and against naming Sedna the tenth planet in our solar system.

D
epending on where you live, you might have seen an aurora in the night sky. You might know this phenomenon better as the northern lights - it's just the same thing!


Aurorae over Denmark. Credit: Sønderborg Amtsgymnasium


Aurora on Jupiter as observed with the STIS instrument on board the Hubble Space Telescope. Credit: NASA, ESA \& John T. Clarke (Univ. of Michigan).
Aurorae are the result of charged particles from the Sun (the solar wind) colliding with the atmosphere of the Earth. They appear in the areas around the two magnetic poles, because the charged particles from the Sun are directed by the Earth's magnetic field to these regions.

So the ingredients for a good aurora are an atmosphere, the solar wind and a sizeable magnetic field.

Hubble has taken beautiful images of the aurorae on Jupiter and Saturn. Do you think we can expect to find aurorae on other planets?

Exercise 4a: Fill out the empty fields of this form to find likely candidates for aurorae: you can make use of the webpages from the Adler's Planetarium on magnetic fields of the planets: http://www.adlerplanetarium.org/learn/planets/planetary_geology/magnetic_fields.ssi

|  | Atmosphere | Magnetic field | Solar wind | Chances of aurora |
| :--- | :---: | :---: | :---: | :---: |
| Mercury |  |  | Strong |  |
| Venus |  |  | Strong |  |
| Earth |  |  | Strong | Excellent |
| Mars |  |  | Strong |  |
| Jupiter |  |  | Medium | Excellent |
| Saturn |  |  | Medium | Excellent |
| Uranus |  |  | Medium to weak |  |
| Neptune |  |  | Weak |  |
| Pluto |  |  |  |  |

When Hubble or the next space telescope goes aurora hunting, which planets would you recommend they start with?


[^0]When two galaxies collide, it's not like a car crash or two billiard balls hitting each other, it is more like interlocking your fingers. Most of the stars in the colliding galaxies will pass unharmed through the collision.

You might wonder why this is. Let us try to make a few calculations as a thought experiment:

How many stars can we pack into the volume of the solar system?

- First we need to agree on what the volume is. Imagine the solar System as thick disk (or a very flat cylinder).
- Let's say the outer limit of the Solar System is the orbit of Pluto (this is an underestimate, since, as you already know there are many other objects out there, including Sedna). This will give us a radius (in km) of $5913.5 \times 10^{6} \mathrm{~km}$.
- Our Solar System is fairly flat, but we can take its thickness (or depth) to be how far Pluto's orbit deviates from the ecliptic - the plane in which all the other planets in the Solar System rotate. This will give us the disk thickness of $1.865 \times 10^{6} \mathrm{~km}$.
- Exercise 5a: Use this data to calculate the volume of the Solar System (in $\mathrm{km}^{3}$ ).
$\qquad$ km ${ }^{3}$

Now to find out how many stars like the Sun we could pack into the Solar System, we now need the volume of the Sun.

- Exercise 5b: The radius of the Sun is 696,000 km. So calculate the volume of the Sun.
$\qquad$ km ${ }^{3}$
- Exercise 5c: Calculate the number of stars of the same volume as the Sun that can fit into the volume of our Solar System.
$\qquad$ stars
- Compare this number with the $2 \times 10^{11}$ stars in the Milky Way.

Now, you might want to conclude that this means that the Solar System is huge. It is not - at least not on the scale of the Universe, or even our galaxy! The diameter of the Solar System is $1 / 100000000$ of the diameter of the Milky Way. The calculation indicates that the galaxy is an incredibly empty place.

And between galaxies space is even emptier.

One of the major achievements of the Hubble Space Telescope has been to detect black holes at the centres of galaxies and to measure their masses. These black holes have turned out to be extremely massive objects. In this exercise, you will measure the black hole mass for the galaxy NGC 4261.

There are various ways to measure the mass of a black hole at the centre of a galaxy. All methods use the fact that any object (e.g. stars or gas clouds) in the vicinity of the black hole moves fast due to the strong gravitational attraction exerted by the black hole.

The galaxy NGC 4261 has a disk of gas and dust at its centre (Figure 1). We will use the velocities of this gas to measure the mass of the black hole at the centre. To figure out how to do this we will first have a look at our own Solar System.


Figure 1: The centre of NGC 4261 with the surrounding disk of dust and gas.
The planets in our Solar System follow approximately circular orbits around the Sun. To keep an object in a circular orbit requires a force that pulls towards the centre of the circle. If such a force were absent the object would fly off into space. The formula for this force $F_{c}$ is:
$\mathrm{F}_{\mathrm{c}}=\mathrm{m} \mathrm{v}^{2} / \mathrm{r}$,
where $m$ is the mass of the planet, $v$ its velocity and $r$ the distance of the circular orbit from the centre. (This should be familiar from Exercise 2). For the Solar System the gravitational force of the Sun provides the attraction that keeps the planets in orbit. The formula for this force $F_{\text {grav }}$ is:
$F_{\text {grav }}=G M_{\text {sun }} m / r^{2}$,
where $M_{\text {sun }}$ is the mass of the Sun.

In other words, $F_{c}$ equals $F_{\text {grav }}$ in the Solar System.

## Exercise 6a

Write down the formula from which you can determine the mass of the sun using $F_{c}=F_{\text {grav }}$. (Look back to Exercise 2 if you need to)

## Exercise 6b

Use the formula found in exercise 6a to determine the mass of the Sun (called solar mass) with the velocities and distances of two of the planets shown in Figure 2. Rewrite the formula from exercise 6a to express the planet velocity as a function of $r$. Plot the curve which represents this function in Figure 2 - you should find a good fit between the planetary points and your curve.


[^1]
## Exercise 6c

The gas in the disk of the galaxy NGC 4261 circles the black hole at the centre of the galaxy in much the same way as the planets orbit the Sun. Figure 3 shows the orbiting velocity of the gas versus its distance from the galaxy centre. Draw a curve by eye in Figure 3 that has a similar shape to the one you drew in Figure 2 that fits the gas velocities. The curve does not have to go exactly through the velocities because these have measurement errors associated with them. Thus, the points will scatter around the curve. Read off velocities and distances at three locations in the curve and determine the mass of the black hole by taking the average and using your formula from Exercise 6b. How many solar masses does this correspond to?


[^2]Astronomers have searched for black holes and determined their masses in just the same way as you have using data from the Hubble Space Telescope. It seems that most and perhaps all galaxies in the Universe harbour a black hole. Moreover, the mass of the black hole at a galaxy centre is an almost constant fraction of the total mass in stars in the host galaxy. This hints at a long and intimate connection between the way the black hole grew in mass and the stars formed in the galaxy. How this connection came about is currently a puzzle to astronomers. It is such puzzles that Hubble's successor, the James Webb Space Telescope, will try to solve.

## References:

Figure 1 was adapted from Verdoes Kleijn et al., 1999, AJ, 188, 2592
The data for NGC 4261 were obtained from Ferrarese, Ford \& Jaffe, 1996, ApJ, 470, 444

Cepheids are bright stars that vary very regularly in brightness. They are named after the star d-Cephei in the constellation of Cepheus. You can see the variation for yourself (if you have a little patience). If you look at the constellation Cepheus over several days, you will see that one of the bright stars changes in brightness every day - that star is d-Cephei.

When we look at the night sky, some stars are brighter than others. Some of the bright stars are really rather small and faint, but just happen to be our next-door neighbours. Others are actually very distant, but they shine with a huge luminosity. It is no simple matter to distinguish between these two cases, and much of the work astronomers do is dedicated to this task. If you know how bright a star is intrinsically, you can deduce the distance to that star - much in the same way as we might judge distance to a car in the night by evaluating the brightness of the headlights subconsciously - much in the same way as we might judge distance to a car in the night by evaluating the brightness of the headlights subconsciously.

The interesting fact about Cepheid variable stars is that the slower they vary, the brighter they are.

So, by observing the period P of a Cepheid variable star, you can find out how bright it really is, and so you can measure the intrinsic brightness $M$. The brightness of stars is measured by a dimensionless number - a number without any units - and the smaller the number, the brighter the star.

The equation connecting the intrinsic brightness $M$ and the period of at Cepheid variable star is called the period-luminosity relation ${ }^{1}$ :
$M=-2.78 \log (P)-1.35$

So from the period, $P$, of a Cepheid variable star you can find the brightness $M$. Use this formula to fill out the brightness column $(M)$ in the table below.

Once we know the brightness we can find its distance using the distance equation that relates the distance $D$ to a star, the intrinsic brightness $M$ of a star and the observed brightness $m$ of a star:
$D=10^{(m-M+5) / 5}$
When you use this equation, the distance is calculated using a unit called the parsec - one favoured by astronomers. If you want to convert the distance to light-years, you just have to multiply the distance by $3.26^{2}$.

- Exercise 7a: Use the distance equation to calculate the distance to the stars and fill out the D column in the table below ( $m$ is of course called m average, as the brightness varies - these stars are all Cepheid variable stars).

| Star number | Period (days) | $\mathbf{m}$ average | $\mathbf{M}$ | D <br> (parsecs) |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 39 | 25.51 |  |  |
| 2 | 29 | 26.49 |  |  |
| 3 | 30 | 26.55 |  |  |
| 4 | 26 | 26.48 |  |  |
| 5 | 25 | 25.55 |  |  |

The Hubble Space Telescope measured the period and the apparent brightness $m$ average for these stars. If you look up the size of the Milky Way you will find out that these Cepheids all lie outside our Milky Way. In fact, the stars belong to the galaxy Messier 100 (M100). You will take a closer look at M100 in the next exercise.

As a side note, you might wonder how astronomers determined that slower varying Cepheids are intrinsically brighter. Obviously, they could not use the period-luminosity relation. Fortunately, there are other techniques to determine distances to stars such as the method of parallax measurements.

Read more about this on the web:
http://curious.astro.cornell.edu/question.php?number=128
or about parallax measurements in general:
http://curious.astro.cornell.edu/question.php?number=237

[^3]As the Universe expands, all galaxies recede from each other. The further away they are from a given observer (e.g. here on Earth), the faster they are receding. It was Edwin Hubble who in 1929 first formulated the expression now known as Hubble's law. It describes the relation between the distance of a given object and its recession velocity, v. Hubble's law is:
$v=H_{0} \times D \quad$ or $\quad H_{0}=v / D$

It states that the galaxies in our Universe are flying away from each other with a velocity, v, proportional to the distance, $D$, between them. $\mathrm{H}_{0}$ is a fundamental property of the Universe - the Hubble constant - important in many cosmological questions and is a measure of how fast the Universe is expanding today.

You might have just calculated the distance to the galaxy M100 in the last exercise. If not, use the accepted value for the distance $-64.7 \times 10^{6}$ lightyears. The recession velocity of M100 is $1400 \mathrm{~km} / \mathrm{s}$.

- Exercise 8a: Convert the distance to M100 from light-years to km, using the speed of light as $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$, and the number of seconds in a year:
$\qquad$ km
- Exercise 8b: Calculate the Hubble constant using the distance to the galaxy M100 and M100's velocity away from us.
$\mathrm{H}_{0}=$ $\qquad$ $\mathrm{s}^{-1}$

The age of the Universe, t , can be approximated by the inverse (or reciprocal) of the Hubble constant $\mathrm{H}_{0}$ - think of the Universe as expanding out approximately constantly - a big assumption - from a point:
$t=1 / H_{0}$

- Exercise 8c: Calculate the age of the Universe using the data from M100. You get the result in seconds - convert it to years.
$\qquad$ years

Astronomers do not agree on a value for the age of the Universe, but somewhere in the vicinity of 15 billion years is not too far from the mark.

There are many challenges when calculating the age of the Universe. Different galaxies do not give the same age. Some areas of the Universe are filled with dust, so the stars seem less bright because of the dust and not because they are far from us.

Even though Cepheid variable stars are bright it is still no small task to measure the variation in brightness of single stars in distant galaxies. The Hubble Space Telescope has been invaluable in measuring Cepheid variables in other galaxies - as part of the Key Project to determine the age and fate of the Universe that is mentioned in the DVD.

## Answers to the exercises

## Exercise 1a

They thought that the Universe consisted of one galaxy, namely the Milky Way. The position of planet Earth was relatively close to the centre of this Universe.

Side note for the teacher: this line of thinking can be viewed as one step in the process throughout history of giving mankind and his Earth a progressively less special and unique place in the cosmos. In ancient times, the Earth was commonly thought to be unique and to reside exactly at the centre of the Universe with all heavenly bodies rotating around it. Copernicus in the 1500 s overthrew conclusively this view placing the Sun at the centre of the Universe. (The principle of assuming that mankind is located at an arbitrary position in time and space in the Universe as opposed to a special one, is often called the Copernican Principle for this reason.) The new view at the beginning of last century discussed here moved the Sun and the Milky Way out of its presumed special place.

## Exercise 1b

The main conceptual changes were:

- The Universe turned out to contain many galaxies instead of one.
- Sun and Milky Way moved from a special place to a arbitrary place in the Universe
- The Universe turned out to be much larger than thought previously.


## Exercise 2a

$6378+568 \mathrm{~km}=6946000 \mathrm{~m}$

## Exercise 2b

$2 \times \mathrm{pi} \times(6378+568 \mathrm{~km})=43643005 \mathrm{~m}$

## Exercise 2c

$96 \times 60=5760 \mathrm{~s}$.

## Exercise 2d

7577 m/s.

## Exercise 2e

$M_{\text {Earth }}=v^{2} \times r / G$

## Exercise 2 f

$6.0 \times 10^{24} \mathrm{~kg}$

## Exercise $\mathbf{2 g}$

Scientists have been using various methods to derive the mass of the Earth: e.g., motion of the Moon, Foucault's pendulum, artificial satellites (e.g., the GRACE project:
http://www.csr.utexas.edu/grace/science/gravity_measurement.html) and motion of falling bodies.

## Exercise 3a

A planet orbits a star. A moon orbits a planet or a larger 'mother-body'.

## Exercise 3b

Surprisingly, there exists no scientific precise definition of the concept of a planet. This allows some freedom to think of Sedna as a planet or not.

Arguments in favor of Sedna being a planet:

- It orbits around the Sun
- It is three-quarters the size of Pluto.

Arguments against:

- Sedna is most likely part of a larger population of "minor planets". We do not consider such populations planets. For example the population of bodies between Mars and Jupiter are called asteriods.
- There are many smaller bodies at distances similar to those of Sedna. Scientists call those "Kuiper Belt objects".


## Exercise 4a

Jupiter and Saturn offer the best chances of succesful observations. (Nevertheless, it might be interesting to study the weaker and/or rarer aurora on other planets. This could test the theory of how auroras are produced and perhaps lead to unexpected results.)

## Exercise 5a

pi $\times\left(5913.5 \times 10^{6}\right)^{2} \times 1.865 \times 10^{6}=2.0 \times 10^{26} \mathrm{~km}^{3}$

## Exercise 5b

$4 / 3 \times \mathrm{pi} \times(696,000)^{3}=1.4 \times 10^{18} \mathrm{~km}^{3}$

## Exercise 5c

$1.4 \times 10^{8}$ stars

## Exercise 6a

$F_{c}=F_{\text {grav }}->=m v^{2} / r=G M_{\text {sun }} m / r^{2}->M_{\text {sun }}=v^{2} \times r / G$

## Exercise 6b

Solar mass $=1.989 \times 10^{30} \mathrm{~kg}$

The plot with the curve should look like this:


## Exercise 6c

Three examples of curves which try to fit the datapoints are shown below:


From top to bottom, the curves are for a black hole mass of $16 \times 10^{38} \mathrm{~kg}, 8 \times 10^{8} \mathrm{~kg}$ and $2 \times 10^{38} \mathrm{~kg}$. This corresponds to $8 \times 10^{8}, 4 \times 10^{8}$ and $1 \times 10^{8}$ solar masses.

More detailed measurements of the mass of the black hole in NGC 4261 yield $4.9 \times 10^{8}$ solar masses.

## Exercise 7a + 7b

| Star number | Period (days) | $\mathbf{m}$ average | $\mathbf{M}$ | D <br> (parsecs) |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 39 | 25.51 | -5.77316 | $1.80564 \times 10^{7}$ |
| 2 | 29 | 26.49 | -5.41547 | $2.40488 \times 10^{7}$ |
| 3 | 30 | 26.55 | -5.45640 | $2.51930 \times 10^{7}$ |
| 4 | 26 | 26.48 | -5.28363 | $2.25281 \times 10^{7}$ |
| 5 | 25 | 25.55 | -5.23627 | $1.43633 \times 10^{7}$ |

## Exercise 8a

$64.7 \times 10^{6} \times 3.0 \times 10^{8} \times(365.0 \times 24.0 \times 60.0 \times 60.0) /\left(1.0 \times 10^{3}\right)=6.1 \times 10^{20} \mathrm{~km}$

## Exercise 8b

$1400 / 6.1 \times 10^{20}=2.3 \times 10^{-18} \mathrm{~s}^{-1}$

## Exercise 8c

$1.0 / 2.3 \times 10^{-18} /(365.0 \times 24.0 \times 60.0 \times 60.0)=1.4 \times 10^{10}$ years


[^0]:    Snapshot series of aurorae on Saturn obtained with the Hubble Space Telescope. Credit: NASA, ESA, J. Clarke (Boston University, USA), and Z. Levay (STSCI).

[^1]:    Figure 2. The velocity of the planets Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune plotted versus their distance from the Sun.

[^2]:    Figure 3. Gas velocities at various distances from the galaxy centre of NGC 4261

[^3]:    ${ }^{1}$ The numbers in this exercise assume that the Cepheids are type I (as they are), and we have for simplicity's sake chosen not to address the problem of dust obscuring the line of sight to the stars.
    ${ }^{2}$ A parsec is defined as the distance at which the Astronomical Unit, the distance between the Earth and the Sun extends an angle of one arc-second on the sky. An arc-second is a sixtieth of a sixtieth of a degree.

